

The set of flexible nondegenerate polyhedra of a prescribed combinatorial structure is not always algebraic

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Abstract

We construct some example of a closed nondegenerate nonflexible polyhedron P in Euclidean 3-space that is the limit of a sequence of nondegenerate flexible polyhedra each of which is combinatorially equivalent to P . This implies that the set of flexible nondegenerate polyhedra combinatorially equivalent to P is not algebraic.

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1. Statement of the main results. A *polyhedron* we call a continuous mapping from a 2-dimensional simplicial complex into Euclidean 3-space that is affine on each simplex of the complex. The image of the complex under such a mapping is called a *polyhedron* too. A polyhedron is *embedded* if the mapping is injective.

The image of a k -dimensional simplex of the simplicial complex is called a *vertex*, *edge*, or *face* of the polyhedron, respectively, for $k = 0, 1$, or 2 . We say that a polyhedron is *nondegenerate* if its every face is a nondegenerate triangle (i. e., if the vertices of the face do not lie on a line).

Two nondegenerate polyhedra have the *same combinatorial structure*, if they are mappings from the same simplicial complex or, equivalently, if there is a one-to-one incidence preserving correspondence between their vertices, edges, and faces.

A polyhedron is *flexible* if its spatial shape can be changed continuously only by changes of its dihedral angles, i. e., so that every face remains congruent to itself during the flex. Otherwise the polyhedron is *rigid*.

Recall that, in Euclidean 3-space there are embedded sphere-homeomorphic flexible polyhedra. Moreover, every flexible (boundary-free) polyhedron preserves the inclosed volume and integral mean curvature during the flex. The reader can learn more about these and other properties of flexible polyhedra from the survey articles [1]–[3] and references given there. Among recent papers on the theory of flexible polyhedra we recommend [4].

One of the main results of this article is

Theorem 1: *In Euclidean 3-space, there is a nondegenerate polyhedron P with the following properties*

- (1) P is the image of a sphere-homeomorphic simplicial complex;
- (2) P is rigid;
- (3) *there is a sequence of nondegenerate polyhedra P_n satisfying*
 - (3a) *for every n , P_n and P have the same combinatorial structure;*
 - (3b) P_n *is flexible for every n ;*
 - (3c) P_n *tend to P as $n \rightarrow \infty$.*

Note that the problem whether the limit of a sequence of flexible convex disk-homeomorphic polyhedra is necessarily flexible was studied, for example, by L.A. Shor [5] and A.D. Alexandrov [6].

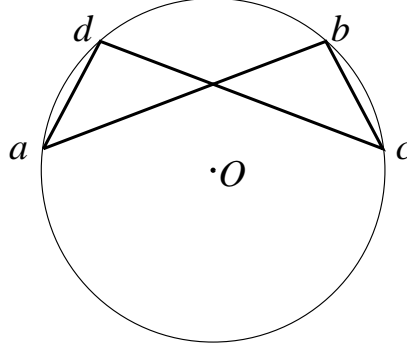


Figure 1: A quadrilateral bar-and-joint framework

They proved that, in some cases, the limit polyhedron is flexible and it is rigid in other cases. To the best of our knowledge, the problem whether the limit of a sequence of flexible sphere-homeomorphic polyhedra is necessarily flexible was never studied before.

In order to formulate one more main result of this article, we fix the notation.

Let Q be a nondegenerate polyhedron in \mathbb{R}^3 . The set of all nondegenerate polyhedra, each of which has the same combinatorial structure as Q has, is denoted by $\llbracket Q \rrbracket$. Since all polyhedra from $\llbracket Q \rrbracket$ are piecewise affine maps of one and the same simplicial complex K , we obtain a “canonical” enumeration of the vertices of each polyhedron from $\llbracket Q \rrbracket$ as soon as we fix an enumeration of the vertices of K . Let v be the number of the vertices of K . Given a polyhedron from $\llbracket Q \rrbracket$, write down the coordinates of all its vertices according to the above “canonical” enumeration. In result, we get some point in \mathbb{R}^{3v} , thus, defining the mapping $\varphi : \llbracket Q \rrbracket \rightarrow \mathbb{R}^{3v}$. Since $\llbracket Q \rrbracket$ consists of nondegenerate polyhedra, its image $\varphi(\llbracket Q \rrbracket)$ is an open subset in \mathbb{R}^{3v} . Denote by $F_{\llbracket Q \rrbracket}$ the set of all flexible polyhedra from $\llbracket Q \rrbracket$.

Theorem 2: *Let P be a polyhedron whose existence is guaranteed by Theorem 1 and let v be the number of vertices of P . There is no an algebraic set $A \subset \mathbb{R}^{3v}$ such that $\varphi(F_{\llbracket P \rrbracket}) = A \cap \varphi(\llbracket P \rrbracket)$.*

Oversimplifying a bit, we can reformulate Theorem 2 as follows: the set $F_{\llbracket Q \rrbracket}$ of flexible nondegenerate polyhedra of a prescribed combinatorial structure is not always an algebraic set. Thus, the set $F_{\llbracket Q \rrbracket}$ of flexible nondegenerate polyhedra is fundamentally different from the set $N_{\llbracket Q \rrbracket}$ of infinitesimally-non-rigid (i. e., admitting nontrivial infinitesimal deformations) polyhedra of a prescribed combinatorial structure. In fact, it is shown in [7]–[8] that there is an algebraic set $A \subset \mathbb{R}^{3v}$ such that $\varphi(N_{\llbracket Q \rrbracket}) = A \cap \varphi(\llbracket Q \rrbracket)$ for every nondegenerate polyhedron Q with v vertices.

The rest of this paper is organized as follows: In Sections 2–4 we construct some auxiliary polyhedra and study their properties that are used in the proof of Theorem 1. The proofs of Theorems 1 and 2 are given in Sections 5 and 6 respectively.

2. Bricard octahedron $B(r)$. First, we construct some auxiliary flexible octahedron B . In literature, it is usually referred to as *Bricard octahedron of type 2* (see, e. g., [9]–[12], [13, pp. 239–240] or [14] and the references therein).

In the plane, consider two straight-line segments ab and bc that do not lie on a line (Fig. 1). Let a point d be symmetric to b with respect to the line perpendicular to the straight-line segment ac and passing through its midpoint. We assume that the four segments ab , bc , cd , and da are rigid and connected with each other at the vertices a , b , c , and d by flexible joints. Obviously, this quadrilateral bar-and-joint framework admits continuous deformations due to changes of the angles between its four rigid straight-line segments, while the points a , b , c , and d are always located in a single plane.

The above assumptions about the bar-and-joint framework allow us to construct a Bricard octahedron of type 2. Nevertheless, in order to prove Theorem 1, it is more convenient to use a Bricard

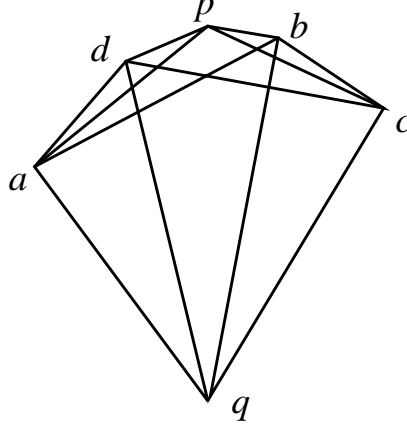


Figure 2: Bricard octahedron $B(r)$

octahedron of type 2 which is subject to the following two additional conditions: During the continuous deformation of the bar-and-joint framework (a) the points a , b , c , and d never lie on a line; (b) the straight-line segments ad and bc are never located on parallel lines. Below we always suppose that conditions (a) and (b) are satisfied although we will not use them before Section 5.

Obviously, d lies on the circle passing through a , b , and c irrespectively of what of the above-described deformations is applied to the quadrilateral bar-and-joint framework. Denote the center and radius of this circle by O and r . (Note that the condition (b) is equivalent to the fact that no one of the straight-line segments ab , cd contains the point O .) Fix some particular value $r = r_0$. Then fix values of the auxiliary parameters s and t such that $t > s \geq r_0$. Throughout this article, the values of the parameters s and t will not change. In the subsequent constructions, we assume that r stands for the radius of the circle passing through the points a , b , and c and satisfies the inequalities $s - \varepsilon < r \leq s$, where ε is a sufficiently small positive number. On the straight line perpendicular to the plane abc and passing through O find the two points $p = p(r)$ and $q = q(r)$ such that p and q belong to different half-spaces determined by the plane abc , the Euclidean distance between p and O is equal to $\sqrt{s^2 - r^2}$, and the Euclidean distance between q and O is equal to $\sqrt{t^2 - r^2}$.

Connect each of the points a , b , c , and d with each of the points p and q by a straight-line segment as it is shown on Fig. 2. In result, we obtain some configuration consisting of 6 points, 12 straight-line segments and the following 8 nondegenerate triangles abp , bcp , cdp , dap , abq , bcq , cdq , and daq . Obviously, this configuration may be treated as a polyhedron denoted by $B = B(r)$ which has the same combinatorial structure as the regular (convex) octahedron. Now we can explain the geometric meaning of the above parameters t and s . They are equal to the lengths of the “side edges” ap , bp , cp , dp and, respectively, aq , bq , cq , dq of the octahedron $B(r)$.

The octahedron $B(r)$ is called *Bricard octahedron* of type 2. The reader may read about Bricard octahedra of types 1 and 3, e.g., in [9]–[12], [13, pp. 239–240] or [14]. In this paper, we avoid presenting their constructions and properties since we do not use them below.

For us, it is important that the octahedron $B(r)$ is flexible. This follows from the fact that the parameter r can be changed arbitrarily within some interval $s - \varepsilon < r \leq s$ while the lengths of all edges of $B(r)$ remain unaltered. More over, $B(r)$ admits only 1-parameter family of nontrivial flexes. The latter means that every continuous deformation of $B(r)$, preserving r and all edge lengths of $B(r)$, is generated by a family of isometric transformations of 3-space.

We will not use any deep properties of 1-parametric polyhedra. The reader, interested in this class

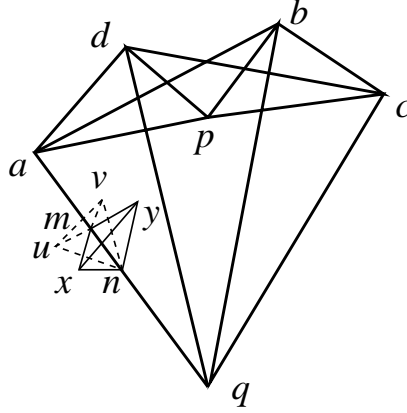


Figure 3: Construction of $C(s)$. Here the point p lies in the plane of the quadrilateral $abcd$

of polyhedra, is referred to [15] and the references therein.

In order to formulate the second property of $B(r)$ which is important for us, we denote by $\alpha(r)$ the value of the smallest of the dihedral angles between the triangles abq and adq and denote by $\beta(r)$ the value of the smallest of the dihedral angles between the triangles adq and dcq . It follows from Fig. 2 that, for $r = s$, the point p lies in the plane passing through the points a , b , c , and d . Moreover, in this case (i. e., for $r = s$) one of the quantities $\alpha(r)$ and $\beta(r)$ attains its maximal value, while the other attains its minimal value. To be specific, we assume that $\alpha(r)$ attains its minimum at $r = s$.

3. Polyhedron $C(r)$. Associate with $B(s)$ the following six points (Fig. 3): m and n internal points of the edge aq ; y , an internal point of the face abq ; v , an internal point of the face adq ; x , a point in the plane of the triangle adq , more precisely, in the open half-plane of this plane that is defined by the line aq and does not contain d ; and u , a point in the plane of the triangle abq , more precisely, in the open half-plane of this plane that is defined by the line aq and does not contain the point b .

Remove the triangle mny from the face abq of $B(s)$ and “glue” the resulting triangular hole by the lateral surface of the triangular pyramid $mnty$. The triangles mny , mty , and nty , that form this lateral surface, are shown in Fig. 3 by thin solid lines. Triangulate the hexagonal face $abqnym$ of the resulting polyhedron by the straight-line segments ay , by , and qy (they are not shown in Fig. 3 to avoid complicating the image).

Remove the triangle mnv from the face adq of the newly constructed polyhedron and “glue” the resulting triangular hole by the lateral surface of the triangular pyramid $mnuv$. The triangles mnu , muv , and nuv , forming this lateral surface, are shown in Fig. 3 by thin solid lines. Triangulate the hexagonal face $adqnvu$ of the resulting polyhedron by the straight-line segments av , dv , and qv (they are not shown in Fig. 3 to avoid complicating the image).

We denote the resulting nondegenerate polyhedron by $C(s)$. Observe the properties of $C(s)$:

(a) The dihedral angle of the tetrahedron $mnuv$ at the edge mn is equal to $\alpha(s)$ (to avoid confusion, declare explicitly that, while all vertices of this tetrahedron are also vertices of $C(s)$, the edge uv of the tetrahedron $mnuv$ is not an edge of $C(s)$).

(b) The polyhedron $C(s)$ is flexible. In fact, we can move its vertices a , b , c , d , p , and q in 3-space exactly in the same way as the same vertices of $B(s)$ move under continuous isometric deformations of $B(s)$. During such a movement, the tetrahedra $mnuv$ and $mnty$ remain congruent to themselves and follow the movements of the faces adq and abq of $B(r)$; the parameter r changes in an interval

$I = (s - \varepsilon, s]$ of the real line ($\varepsilon > 0$). Denote by $C(r)$ the polyhedron from the above-described continuous family of polyhedra that corresponds to the particular value of $r \in I$.

(c) For $r \in (s - \varepsilon, s)$, the dihedral angle $\gamma(r)$ of the tetrahedron $mnux$ at the edge mn is strictly greater than $\alpha(s)$: $\gamma(r) > \alpha(s)$. In fact, $\gamma(r) = \alpha(r) > \min_{r \in (s - \varepsilon, s)} \alpha(r) = \alpha(s)$.

4. Polyhedron P . Continue constructions of Section 3. Denote by w the midpoint of the straight-line segment mn . Let h_k be a homothetic transformation of 3-space with center w and scale factor $k > 0$. The latter is supposed to be so small that the image $h_k(aq)$ of the straight-line segment aq is contained in the straight-line segment mn : $h_k(aq) \subset mn$. Further, let T be the rotation of 3-space around the straight line mn to the angle π . Denote by $B'(s)$ the image of the octahedron $B(s)$ under the action of the mapping $T \circ h_k$. Similarly, put by definition $z' = (T \circ h_k)(z)$ for every vertex z of $B(s)$.

Diminishing, if need be, the scale factor k , we assume that d' lies inside the face mnx and b' lies inside mnv (see Fig. 3).

At last, remove the triangles $a'b'q'$ and $a'd'q'$ from the union of $C(s)$ and $B'(s)$. (Recall that, in this paper, a “polyhedron” means a “polyhedral surface”, rather than a “solid body.”) Denote the resulting polyhedron by P .

In Section 5 we prove that P may serve as a polyhedron whose existence is proclaimed in Theorem 1.

Now we study possible continuous isometric deformations of P . In this study, we compare deformations of some sets of vertices of P (e.g., $\{a, b, c\}$), with deformations of the corresponding sets of vertices of the octahedron $B(r)$ (here we mean $B(r)$ as it was constructed in Section 2 when it had nothing in common with P). In order to distinguish these two cases, we write, for example, $\{a, b, c\}_P$ and $\{a, b, c\}_{B(r)}$ respectively.

Lemma: *Let P be subject to a continuous isometric deformation such that each of its vertex is sufficiently close to its original position. Let Q be the result of such deformation and let its vertices are denoted by the same letters as the corresponding vertices of P (see, e.g., Fig. 3). Then there is $r \in I$ such that the set $\{a, b, c, d, p, q\}_Q$ of vertices of Q is congruent to the set $\{a, b, c, d, p, q\}_{B(r)}$ of the same vertices of $B(r)$.*

Proof: Fix $r \in I$ so that the Euclidean distance between the vertices a and c of Q (see Fig. 3) is equal to the Euclidean distance between the vertices a and c of $B(r)$. Then the tetrahedron $\{a, b, c, p\}_Q$ is congruent to the tetrahedron $\{a, b, c, p\}_{B(r)}$ because their corresponding edges have the same length. Similarly, $\{a, b, c, q\}_Q$ is congruent to $\{a, b, c, q\}_{B(r)}$.

Observe that the tetrahedra $\{a, b, c, p\}_Q$ and $\{a, b, c, q\}_Q$ are attached to each other along the nondegenerate (i.e., not lying on a line) triangle abc (see the condition (a) in Section 2). Moreover, they lie in the different half-spaces determined by the plane abc . The latter is true because, when we constructed the points p and q in Section 2, we assumed that p and q lie in the different half-spaces determined by the plane abc . Similarly, $\{a, b, c, p\}_{B(r)}$ and $\{a, b, c, q\}_{B(r)}$ are attached to each other along abc and lie in the different half-spaces determined by the plane abc . Hence, the sets $\{a, b, c, p, q\}_Q$ and $\{a, b, c, p, q\}_{B(r)}$ are congruent:

$$\{a, b, c, p, q\}_Q \cong \{a, b, c, p, q\}_{B(r)}. \quad (1)$$

Obviously, the above argument remains valid if we replace the points a, b , and c with a, b , and d . Hence,

$$\{a, b, d, p, q\}_Q \cong \{a, b, d, p, q\}_{B(r)}. \quad (2)$$

From (1) and (2) it follows

$$\{a, b, c, d, p, q\}_Q \cong \{a, b, c, d, p, q\}_{B(r)}, \quad (3)$$

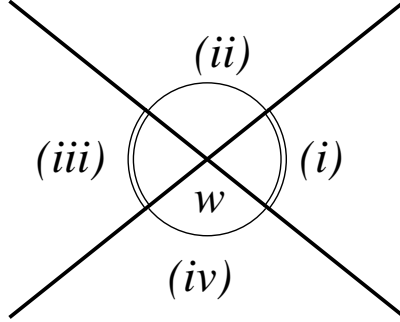


Figure 4:

provided that a, b, p , and q do not lie in a plane. The latter condition is fulfilled because the straight-line segment ab does not pass through the center of the circumscribed circle of the quadrilateral $abcd$ (see condition (b) in Section 2).

Thus, (3), as well as the lemma, is proved.

Informally, the meaning of the lemma can be explained as follows (see Fig. 3). When constructing the polyhedron P , we removed the straight-line segment mn from the edge aq . This means that, in principle, the straight-line segments am and nq do not have to lie on a straight line during an isometric deformation of P . Moreover, when constructing the polyhedron P , we triangulated the faces $amynqb$ and $amvnqd$. This means that, in principle, the triangles of the triangulation of $amynqb$ (as well as of $amvnqd$) do not have to lie in a single plane during an isometric deformation of P . The lemma just shows that this does not happen, i.e., that, for every isometric deformation of P the straight-line segments am and nq necessarily lie on the line aq , all triangles of the triangulation of $amynqb$ lie in the plane abq , and all triangles of the triangulation of $amvnqd$ lie in the plane adq .

5. Proof of Theorem 1. By construction, P is the image of a sphere-homeomorphic simplicial complex. Hence, property (1) is satisfied.

The fact that P does not admit nontrivial isometric deformations is nearly obvious because we obtained P by gluing a dihedral angle of the octahedron $B'(s)$ to a dihedral angle of the flexible polyhedron $C(s)$ “from the outside”. We already know that each of these dihedral angles may only increase during isometric deformation. On the other hand, the sum of these dihedral angles is equal to 2π . Hence, each of the dihedral angles remain constant during every isometric deformation. This implies that every isometric deformation of P is trivial.

Let us explain the above in more detail.

Fix some isometric deformation of P and prove that this deformation is trivial.

Consider the four dihedral angles of $B'(s)$ that meet at the edge $a'q'$. (To be more precise, we should say that here we are talking only about those vertices, edges, and faces of $B'(s)$ that became parts of P during the above-described construction of P .) For clarity, in Fig. 4 we show the section of a small neighborhood of the point w by the plane passing through w perpendicular to the straight line $a'q'$. In Fig. 4, (i) stands for the section of the dihedral angle of the tetrahedron $m n x y$ at the edge mn , (ii) stands for the section of the dihedral angle of the octahedron $B'(s) \subset P$ at the edge $a'q'$, (iii) stands for the section of the dihedral angle of the tetrahedron $m n u v$ at the edge mn , and (iv) stands for the section of the dihedral angle of the polyhedron $B(s) \subset P$ at the edge aq .

Observe that two of these dihedral angles, denoted by (i) and (iii) in Fig. 4, are dihedral angles of the tetrahedra $m n x y$ and $m n u v$ at the edge mn . Hence, the values of these dihedral angles remain unaltered during any isometric deformation of P .

The dihedral angle, denoted by (ii) in Fig. 4, is the dihedral angle of the octahedron $B'(s)$ at the edge $a'q'$ and is equal to $\alpha(s)$. By the choice of s , this angle can only increase during every isometric deformation of $B'(s)$ (and, hence, during every isometric deformation of P).

At last, the fourth of the dihedral angles under consideration, denoted by (iv) in Fig. 4, is the dihedral angle of the octahedron $B(s)$ at the edge aq . This angle is equal to $\alpha(s)$ also. According to the lemma from Section 4, for every isometric deformation of P , this dihedral angle is equal to the dihedral angle $\alpha(r)$ of the octahedron $B(r)$ for some particular value of the parameter r . Hence, the angle (iv) can only increase during any isometric deformation of P .

On the other hand, the sum of the four angles (i)–(iv) is constant during any isometric deformation of P and is equal to 2π . Hence, each of the angles remains constant and the deformation is necessarily trivial. Thus, property (2) from the statement of Theorem 1 is satisfied.

Now we will construct a polyhedron P_n which appeared in (3) of Theorem 1. We construct P_n as a modification of P . (Recall that, when we constructed the polyhedron P in Section 4, we put $r = s$.) More precisely, we left unaltered all vertices of P but p , all edges of P but the four edges ap , bp , cp , and dp , and all faces of P but the four faces abp , bcp , cdp , and adp . We replace the vertex p of P by a new vertex p_n so that the length of each of the four new edges ap_n , bp_n , cp_n , and dp_n is equal to $s + 1/n$. The resulting polyhedron is denoted by P_n .

Obviously, P_n and P have the same combinatorial structure and $P_n \rightarrow P$ as $n \rightarrow \infty$. Moreover, P_n is flexible for every n , since the points a, b, c, d , and p_n do not lie in a plane and the angle (iv), shown in Fig. 4, can be diminished. Hence, (3) is satisfied and Theorem 1 is proven.

6. Proof of Theorem 2. We argue by contradiction. Suppose there is an algebraic set $A \subset \mathbb{R}^{3v}$ such that $\varphi(F_{\llbracket P \rrbracket}) = A \cap \varphi(\llbracket P \rrbracket)$.

Since A is closed in \mathbb{R}^{3v} , the set $\varphi(F_{\llbracket P \rrbracket})$ is relatively closed in $\varphi(\llbracket P \rrbracket)$.

On the other hand, if P_n is the polyhedron from the statement of Theorem 1 then $\varphi(P_n) \in \varphi(F_{\llbracket P \rrbracket})$ for all n ; moreover, $\varphi(P_n) \rightarrow \varphi(P)$ as $n \rightarrow \infty$ and $\varphi(P) \in \varphi(\llbracket P \rrbracket)$, but $\varphi(P) \notin \varphi(F_{\llbracket P \rrbracket})$. Hence, $\varphi(F_{\llbracket P \rrbracket})$ is not relatively closed in $\varphi(\llbracket P \rrbracket)$.

This contradiction proves Theorem 2.

References

- [1] R. Connelly, Conjectures and open questions in rigidity. In the book: *Proc. Intern. Congr. mathematicians (Helsinki, 1978)*, Vol. 1. Helsinki: Acad. Sci. Fennica, 1980. P. 407–414. MR0562634 (82a:51009), Zbl 0434.53042.
- [2] J.-M. Schlenker, *La conjecture des soufflets (d’après I. Sabitov)*. Astérisque **294** (2004), 77–95. (Bourbaki seminar. Volume 2002/2003. Exposes 909–923. Paris: Société Math. de France). MR2111640 (2005k:52041), Zbl 1078.52014.
- [3] I.Kh. Sabitov, *Algebraic methods for solution of polyhedra*. Russ. Math. Surv. **66**, no. 3 (2011), 445–505. MR2859189 (2012k:52027), Zbl 1230.52031.
- [4] A.A. Gaifullin, *Sabitov polynomials for volumes of polyhedra in four dimensions*. Adv. Math. **252** (2014), 586–611. MR3144242, Zbl 1301.51027.
- [5] L.A. Shor, *On flexibility of convex polyhedra with boundary*. Mat. Sb. **45**, no. 4 (1958), 471–488. (In Russian; English translation is available in [6] as Appendix 3.) MR0099062 (20 #5506), Zbl 0088.14502.
- [6] A.D. Alexandrov, *Convex polyhedra*. Berlin: Springer, 2005. MR2127379 (2005j:52002), Zbl 1067.52011.

- [7] Eh.G. Poznyak, *Infinitesimally non-rigid closed polyhedra*. Vestn. Mosk. Univ., Ser. I **15**, no. 3 (1960), 14–18. (In Russian.) MR0131814 (24 #A1662), Zbl 0098.35404.
- [8] H. Gluck, *Almost all simply connected closed surfaces are rigid*. Lect. Notes Math. **438** (1975), 225–239. MR0400239 (53 #4074), Zbl 0315.50002.
- [9] R. Bricard, *Mémoire sur la théorie de l’octaèdre articulé*. J. Math. **3** (1897), 113–148. JFM 28.0624.01.
- [10] H. Lebesgue, *Octaèdres articulés de Bricard*. Enseign. Math., II Sér. **13** (1967), 175–185. MR0234347 (38 #2664), Zbl 0155.49301.
- [11] H. Stachel, *Zur Einzigkeit der Bricardschen Oktaeder*. J. Geom. **28**, no. 1 (1987), 41–56. MR0878483 (88c:51021), Zbl 0611.51011.
- [12] I.Kh. Sabitov, Local theory on bendings of surfaces. in the book: *Geometry III. Theory of Surfaces. Encycl. Math. Sci.*, **48** (1992), Berlin: Springer. P. 179–250. MR1306736 & MR1039820 (91c:53004), Zbl 0711.53002.
- [13] P.R. Cromwell, *Polyhedra*. Cambridge: Cambridge Univ. Press, 1999. MR1458063 (98j:52017), Zbl 0926.52014.
- [14] V. Alexandrov, *The Dehn invariants of the Bricard octahedra*. J. Geom. **99**, no. 1–2 (2010), 1–13. MR2823098 (2012h:52032), Zbl 1231.52019.
- [15] I.G. Maksimov, *Description of the combinatorial structure of algorithmically 1-parametric polyhedra of spherical type*. Sib. Math. J. **53**, no. 4 (2012), 718–731. MR3013534, Zbl 1260.52015.

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